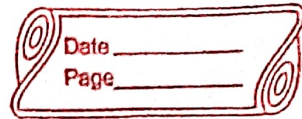


16. 02. 2024



## Linear Differential Equations

1. Solve  $\frac{d^4 y}{dx^4} - y = x \sin x.$

Soln. For CF  $\left(\frac{d^4}{dx^4} - 1\right) y = 0$

$$\Rightarrow D^4 - 1 = 0$$

$$\Rightarrow (D^2 + 1)(D^2 - 1) = 0 \Rightarrow (D+1)(D-1)(D+i)(D-i) = 0$$

$$\Rightarrow D = i, -i, 1, -1$$

$$\therefore CF = c_1 e^{ix} + c_2 e^{-ix} + c_3 e^x + c_4 e^{-x}$$

$$= A \cos x + B \sin x + c_3 e^x + c_4 e^{-x} \quad \text{--- (1)}$$

$$\left[ \because c_1 e^{ix} + c_2 e^{-ix} = c_1 (\cos x + i \sin x) + c_2 (\cos x - i \sin x) \right.$$

$$= \underbrace{(c_1 + c_2)}_A \cos x + i \underbrace{\sin x (c_1 - c_2)}_B$$

$$\text{Let } A = c_1 + c_2 \text{ and } B = i(c_1 - c_2)$$

$$\therefore = A \cos x + B \sin x.$$

For PI

$$PI = \frac{1}{(D^4 - 1)} x \sin x$$

$$\therefore PI = \frac{1}{(D^2+1)(D^2-1)} x \sin x$$

Important step

$$\Rightarrow PI = \text{Imaginary part of } \frac{1}{(D^2+1)(D^2-1)} x (\cos x + i \sin x)$$

$$= \text{I.P. of } \frac{1}{(D^2+1)(D^2-1)} e^{ix} x$$

$$= \text{I.P. of } e^{ix} \frac{1}{(D+i)^2 - 1} x$$

[ D के स्थान पर D+i ( $= D+i$ )  
 $e^{ix} = e^{ax}$   
 $\therefore a = i$  ]

$$\Rightarrow PI = \text{I.P. of } e^{ix} \frac{1}{D^4 + 4D^3 + 6D^2 + 4Di + 1 - 1} x$$

[ Binomial theorem से expand करें ]

$$(a+x)^4 = a^4 + 4C_1 a^3 x + 4C_2 a^2 x^2 + 4C_3 a x^3 + x^4$$

$$\therefore PI = \text{I.P. of } e^{ix} \frac{1}{D^4 + 4D^3 - 6D^2 - 4Di + 1} x$$

$$\Rightarrow PI = \text{I.P. of } e^{ix} \frac{1}{-4Di \left[ 1 - \left( \frac{6D^2 - 4D^3 - D^4}{4Di} \right) \right]} x$$

$$\Rightarrow PI = \text{I.P. of } e^{ix} \frac{1}{-4Di} \left[ 1 - \left( -\frac{3D}{2}i - D^2 + \frac{1}{4}D^3i \right) x \right]$$

$$PI = \text{I.P. of } e^{ix} \frac{i}{4D} \left[ 1 + \left( \frac{3D}{2}i + D^2 - \frac{1}{4}D^3i \right) x \right]$$

$$\Rightarrow PI = \text{I.P. of } e^{ix} \frac{i}{4D} \left[ 1 - \left( \frac{3D}{2}i + D^2 - \frac{1}{4}D^3i \right) + \text{higher powers of } D \right] x$$

$$= \text{I.P. of } \frac{ie^{ix}}{4D} \left[ x - \frac{3ix}{2} \right]$$

$$= \text{I.P. of } \frac{ie^{ix}}{4} \int \left( x - \frac{3}{2}i \right) dx$$

$$= \text{I.P. of } \frac{ie^{ix}}{4} \left[ x^2 - \frac{3ix}{2} \right]$$

$$= \text{I.P. of } \frac{1}{4} \left[ i \cos x - \sin x \right] \left( x^2 - \frac{3ix}{2} \right)$$

$$\Rightarrow PI = \frac{1}{4} \cos x \left( x^2 - \frac{3ix}{2} \right) \quad \text{--- (2)}$$

$\therefore$  complete solution is given by

$$y = CF + PI$$

where CF is given by eq (1)

and PI is given by eq (2).